PEDAGOGIC ACTIONS AND STRATEGIES THAT SUPPORT SOPHISTICATION WITHIN FOUNDATION PHASE NUMBER WORK

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INTRODUCTION

A number of analysts describe the low performance of South African students in mathematics as 'a crisis' (Schollar, 2009; Fleisch, 2007; Van der Berg and Louw, 2006). Van der Berg and Louw (2006) emphasize South Africa's poor scores in mathematics tests compared to other countries in Africa. They also argue that tests scores obtained by South African students on international tests are much lower than those obtained by students of the same age in France, Hong Kong, Singapore and South Korea (Van der Berg & Louw, 2006). Ensor et al. (2009) and Fleisch (2007) note that international studies show South Africa's performance for learners in numeracy is lower than that of eleven other African countries. Concerning the Foundation Phase more specifically, there is evidence of a lack of shift from concrete counting-based strategies to more abstract calculation-based strategies (Ensor et al., 2009) with Schollar (2009) indicating the prevalence of concrete counting strategies well into the Intermediate Phase.

Wright et al (2006) suggest that children need to move towards more sophisticated strategies within work on early number. Other researchers describe this shift in terms of supporting children to move to more abstract models and strategies within early number (Ensor et al., 2009; Fleisch, 2008; Schollar, 2009). In this paper, drawing from my Masters study and literature on early number teaching, linked with findings of early number teaching in South Africa, I present a summary of pedagogic actions and strategies that can be used with common resources to support progression.

Concrete/abstract strategies

Concrete counting-based strategies refer to actions where the learner cannot find the answer to a mathematical problem without using concrete objects. This means that learners cannot solve problems without concrete counting, drawing tallies or using perceptual strategies such as feeling or seeing items. In contrast, abstract calculation-based strategies involve strategies where the child does not need concrete objects to find the answer, but can instead use mental calculations in which numbers have been transformed into abstract entities that can be operated on. Ensor et al. (2009) found that concrete methods to solve problems, such as tally counting, were dominant in the Western Cape schools they investigated through classroom-based research. They argued that these learners' poor mathematical results were the result of insufficient moves from counting to calculating.

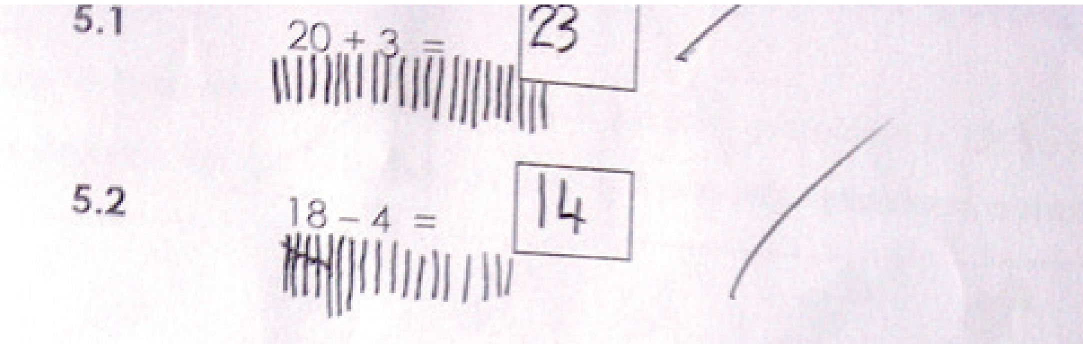
Sfard (1992), writing more theoretically, states that "the term abstracting is commonly used with reference to the activity of creating concepts that do not refer to tangible, concrete objects" (p. 111). A more practical explanation of abstraction is provided by Haylock & Cockburn (2008, p. 42): "when a child adds 3 and 4, what actual objects are, makes no difference to the mathematical process: 3 sweets and 4 sweets, 3 boys and 4 boys, 3 counters and 4 counters, these are all presented in the same abstraction 3+4=7". A child who can transfer the '3 counters and 4 counters' task to her fingers has a more abstract view of numbers than the child who can only count by seeing or feeling counters..

Objectification

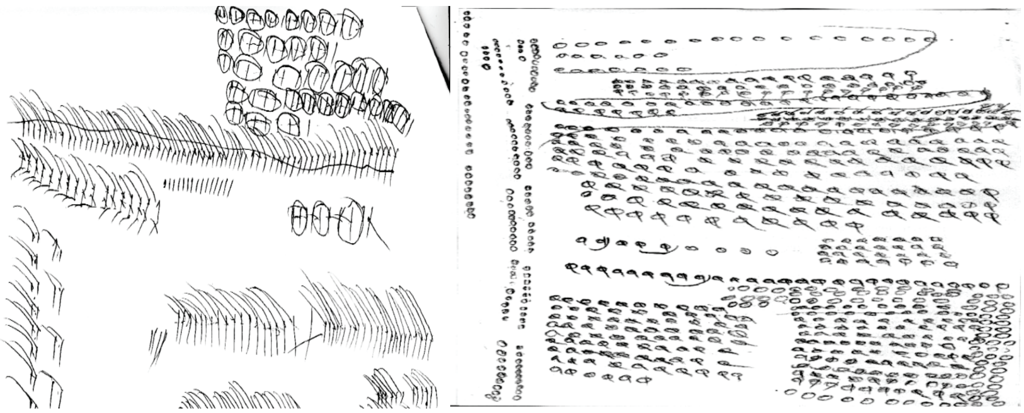
To have an abstract understanding of number, a child should be able to appreciate the quantities underlying numerical symbols without the need for counting. Sfard (2010) argues that *abstract objects* are outcomes of *reification*. The basic principle of reification is that the operational (process-orientated) conception emerges first and over time, becomes reified into the mathematical object (structural conception) (A Sfard & Linchevski, 1994). The *theory of reification* refers to the turning of processes into objects. Sfard (1991) argues: "It seems therefore, that the structural approach should be regarded as a more advanced stage of concept development. In other words, we have good reason to expect that in the process of concept formation, operational conceptions would precede the structural". Sfard (1992) also draws attention to the fact that process and object are inseparable: they are different facets of the same thing. If we look at number sense in the way Sfard's (1992) reification theory suggests, we can say that concrete counting is a way of understanding numbers as a process. When a learner understands number in a more abstract way, (i.e. the number exists without needing to be being associated with concrete counting actions), the learner has a structural understanding of numbers and reification has taken place. Sfard (2008) argues: "Such an act of reification - of discursively turning processes into objects - is the beginning of *objectification,* which, if completed, will leave us convinced about the mind-independent, 'objective' existence of the object-like referent" (p. 44). Sfard (2008) states that objectification is the process which involves "... two tightly related, but not inseparable discursive moves: *reification*, which consist in substituting talk about actions with talk about objects, and *alienation*, which consists in presenting phenomena in a impersonal way, as if they were occurring of themselves, without the participation of human beings" (p. 44). The author argues that objectification builds efficiency in mathematical communication because a learner/teacher can refer to mathematical objects and does not need to describe them in processes terms. Ensor et al. (2009) showed in their study that most of their learners still used concrete methods to find answers to mathematical problems in Grade 3. When children use concrete methods to find the answer, this shows that they have not yet objectified numbers, and remain reliant instead on counting processes.

Example of a process conception of number

In my Masters study, learner responses on the Grade 1 ANA indicated examples of process conceptions of number. A grade 2 child used tallies to find the answer to 20+3= __. His working, detailed below, shows that he made 20 tallies and then another 3 tallies and, counting these together, got the correct answer.



The question was allocated 1 mark for a correct answer, which he was awarded. In writing emanating from this study (Weitz & Venkat, 2013) we noted that mark allocations like these in the early grades are likely to work against persuading teachers to encourage the development of more sophisticated strategies within counting, and early addition and subtraction tasks. In the paper, we argued that a child who understands the 'tens-based' number structure and the meanings that can be associated with addition and the counting number sequence, would know that 20+3 should be 23 without drawing tallies. In this working, neither 20 nor 3 exist as an abstract concept – both numbers only exist as outcomes of concrete unit counting processes. We also noted that the low number range in the Grades 1 and 2 ANAs makes concrete methods feasible as means for evaluating answers to problems, but the increasing number ranges in the curriculum for Grade 3 and beyond, makes ongoing use of unit counting highly cumbersome and error-prone, as Schollar's (2009) report shows:



While many researchers emphasize the need for the shift from concrete to abstract number conceptions, Wright et al. (2006) developed a framework where this shift is described in fine nuances, with stages of progression. Their model, developed from research done by Steffe and colleagues (for example: Steffe, 1992; Steffe and Cobb, 1988; Steffe et al., 1983), focuses particularly on developing children's strategies for solving number problems. Focusing centrally on counting, addition and subtraction, Wright et al. (2006) see counting as a developmental process which they break down into 6 stages: emergent; perceptual; figurative-counting; initial number sequence; intermediate number sequence; and facile number sequence. These stages are referred to as the Stages of Early Arithmetical Learning (SEAL). The details of each stage below underscore that the focus is not simply on whether the child can count, but also the strategies that the child uses to count.

These SEAL stages take the shift from concrete to abstract into account, and break it down to a more detailed set of developmental indicators, and are detailed below:

Perceptual Counting, (stage 1 of SEAL), refers to a child counting by seeing or feeling objects. This is a concrete way of dealing with number. A child on this stage cannot yet associate his fingers with the numbers 4 and 5 to find the answer to 4+5; instead concrete counting objects are required.

In Figurative Counting (stage 2), a child can count unseen objects, but starts at one when she is asked to add screened objects ('screened objects' are objects that are not visible to the learner). In this stage, the child has awareness of 4 and 5 as transferable to their fingers. At this stage she will count out 4 on one hand and then count out five on the other hand and then count them all from 1-9 keeping track with her fingers. Stages 1 and 2 use the 'Triple count method' (Anghileri, 2006)

At Stage 3 which is the Initial Number Sequence, a child uses counting-on strategies rather than count-all strategies. At this stage, the child can use 'count-down-from' strategies to solve removed items tasks, but cannot use 'count-down-to' strategies to solve missing subtrahend tasks.

In the Intermediate Number Sequence stage (stage 4), the child can additionally count down to as well as count down from and can choose the more efficient strategy for particular addition and subtraction problems. For a subtraction task like 11-3, counting-down-from-11 and removing 3 is more efficient, but for 11 - 7 count-on-from-7 or count backwards to 7 is more efficient. If we set the following problem for 5 - ? = 3 (missing subtrahend) in a problem context, a child on stage 4 knows that she can count down from 5 and say 4, 3 ... and state the answer of 2. A child at stage 3 would not be able to answer this question, however, she would be able to count down from 5 to answer the question 5 - 3 (removed items task) and say 4, 3, 2 … also reaching the answer of 2. A child at stage 2 would not be able to do either the removed item task or the missing subtrahend task.

The last stage is Facile Number Sequence stage (stage 5) where a child can use a range of procedures flexibly to find the answer without counting by one. A child at this stage would be able to answer the problem: 5 - ? = 3 and 5 - 3 immediately, as recalled facts or using strategies based on inverse operations, 5 and 10 based number bonds, commutativity or compensation. A child at stage 5 is at the top of the SEAL 'ladder' concerning abstraction of number.

In my study I saw that the majority of learners were on stage 1 or 2 of the LFIN framework which means that they were on the count-all stages.

Pedagogic actions to promote abstract understanding

In the remaining sections of this paper, I consider progression from concrete counting to abstract number objects in relation to specific resources and associated pedagogic actions. Within this discussion, I refer to resources that are increasingly available in the Gauteng classrooms that the broader Wits Maths Connect – Primary project is operating. My motivation for detailing these pedagogic actions with attention to the use of resources within these actions rests on evidence showing poor progressions within teachers' mediation of addition and subtraction tasks even when using structured resources (Venkat & Askew, 2012). This leads to their argument that:

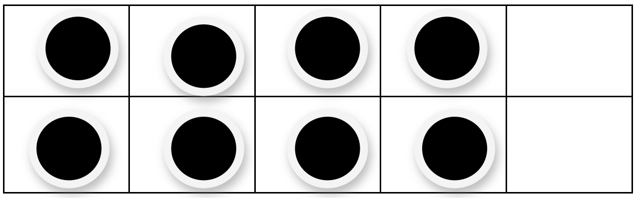
> provision of artefacts that support more abstract ways of working with number is insufficient in improving teaching and learning. (p84)

Their argument follows from artefacts that were used with unit counting and therefore without reference to the 10-base structure of the abacus and the 100 square. There is also broader evidence of disconnections (Venkat & Adler, 2012) and ambiguity and highly 'localized' teacher talk in Foundation Phase classrooms (Venkat & Niadoo, 2012). My descriptions and discussions are therefore directed towards teacher development activities.

Overall, there is evidence that supporting teachers to use resources in ways that model progression, is needed. The table below shows a summary of key actions that teachers should be aware of when using these more structured resources, if they are aiming to support progression to more sophisticated strategies. I will discuss three structured resources and how they can be used to promote more sophisticated number strategies.

Ten-frame cards

Ten-frame cards can be used to show 'odd' and 'even' number arrangements and to count through five. In the 'pair-wise' arrangement below, the aim is that children can 'see' that 2 is missing from the ten-frame card and that the number of dots is 2 less than ten which is 8, and not count the dots one-by-one up to 8.



A 20-Bead-string (in multiples of 5)

20 beads (or bottle tops) with two different colours like below

The aim is to encourage children to show different numbers of beads with single movements. The bead string can also be used to 'add through ten'. Example: 7+5=?: use the bead string to 'find 7' by shifting 5 and 2 beads with one movement.

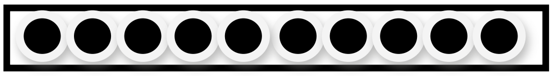
Then say that we know that it is 3 more to get to 10, which involves decomposing 5 into 3+2 and first add 3.

The last movement is the movement of two beads, because 5=3+2

Encourage children to 'see' that the answer on the bead string is 10+2=12, without insisting that they must count out the answer one-by-one.

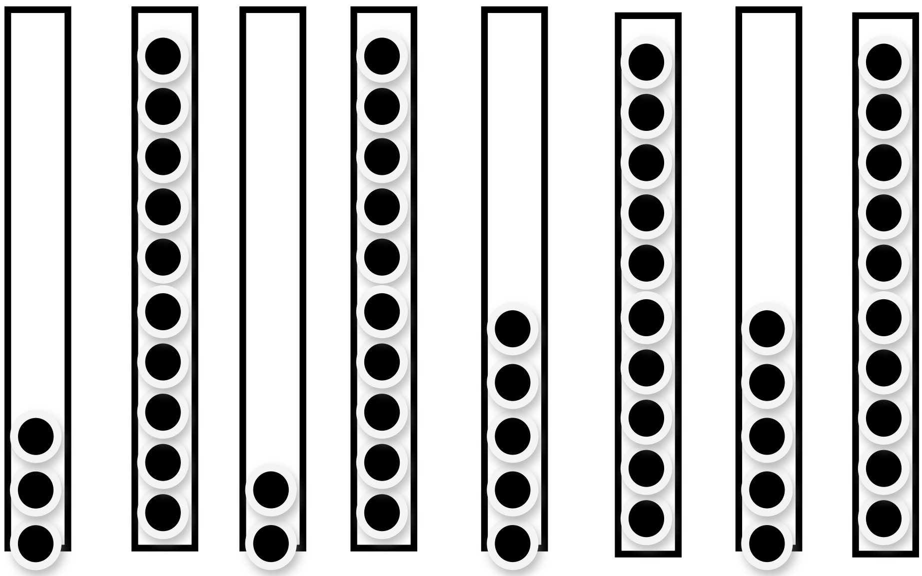
Ten-strip cards

Ten-strip cards can be used to count-on and count-backwards in 10s, either ‘on’ or ‘off’ the multiples of ten. Teaching should establish first, by counting if needed, that one strip, as shown below, has ten dots – making it a ’10-strip’.



Children must be encouraged to work with ten as an objectified quantity rather than needing to count in ones each time. Tasks with 10-strips involve laying out dot strips one at a time with the question: ‘How many dots?’ Encourage learners to count on from the previous total rather than reverting to the start each time.

Example:



3+	10+	2+	10+	5+	10+	8+	10	= __?___
	(=13)	(=15)	(=25)	(=30)	(=40)	(=48)	(=58)	

The table below contrasts more concrete strategies with more abstract strategies in relation to each of these resources to highlight the differences.

Resource	Concrete action/strategy	More abstract action/strategy
10 frame cards	Counting the 8 dots on the cards using unit counting	'See' that there are two open spaces on the 10 frame card
Dot/domino cards	Counting the numbers on the cards using unit counting	Subitize the number of dots by seeing the number without counting them one by one requires familiarity with common dot and dice arrangements and grouped random arrangements
Bead string 1-20	Counting numbers in the 1-20 range using a unit count of beads	Demonstrating counting 'through 5' or 'through 10' with a single movement action
10 Strip cards	Counting the dots on the cards using unit counting	Adding the dots on a 10 strip card as a unit of 10 without counting them again

While above I have highlighted resources and pedagogic actions with them, in classrooms resources are not used in isolation of talk and learner response. Thus, to conclude this paper, I present a case of one learner's response to a set of early number tasks. I then detail the ways in which follow-up tasks with resources, can be used following diagnosis of the learner's SEAL stage, to help the learner to push forward. Thandeka is a pseudonym.

Thandeka

Thandeka can say the Forward Number Word Sequence from 1 to 100. When asked to count on from 76 she skipped 77 and proceeded to 84. She counted on from 94-109 and then said 200, 201, 202, 203, 204, 205 and stopped when she was asked to stop. She can say the Number Word After (NWA) and Number Word Before (NWB) and for numbers 1-15 but not beyond that. Thandeka always counts from one when she adds two numbers. She cannot count backwards beyond 15. She can answer addition tasks where the sum is smaller than 10 but cannot answer 9+6. When she was asked to add 9+6 screened counters, she counted out 9 on her fingers starting with 1 and tries to count-on and then gives answer of 16. When asked to add 8 and 5 screened counters, she gave the answer 18. When asked to find the answer to 16-12 she asked to use the counters. Counting out 16 counters, she then she took away 12 counters one-by-one. She then counted the remaining 4 from 1. She could subitize small single numbers. When the domino cards had 2 sides she had difficulties keeping track of the number of dots without counting them one-by-one, even when the numbers were smaller than 5.

SEAL stage 2- Figurative Counting Stage

Follow-up pedagogic action:

1. To help her to count backwards beyond 15, Thandeka can be asked to repeat short BNWS initially using a number track after the teacher. Example: 'Say after me 18, 17, 16, 15, 14 pointing on the number track. Another way of helping her to count backwards is to say alternate number words backwards in the range 1-30, Example: teacher says 30, child says 29, teacher says 28, child says 27 ext.
2. To help her to subitize numbers of dots, the teacher must develop knowledge and more confidence with dice patterns 1-6 (subitising) by showing the domino cards repeatedly. To subitize the number of dots she has to 'see' the number of dots without counting them one-by-one. This requires familiarity with common dot and dice arrangements and grouped arrangements.
3. She is unable to use count on, to add numbers, so a first step would be to count-on in ones on the bead string and then moving to counting through ten.
4. To help her to add 9+6 the bead string can be used by counting through ten: example 9+1=10. Decompose 6 to 1+5, and then 10+5=15. Another way to help her is to show combinations to 20 on a 20-bead-string

CONCLUSION

Looking at the above, we know that children have to shift their thinking from a concrete understanding of numbers to an abstract way of working with numbers. Teachers must not always bring the children back to unit-counting if the child already made the shift and are ready to move to a next level of abstraction. Progression in number sense is possible and teachers need to focus intentionally to help children to think more abstractly about numbers.

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